CHAPTER 5

MAKING WAVES

We have seen how massive objects can bend space-time, creating a local dimple in the Universe. But what sort of objects create waves in space, and how do they do it? To answer these questions, let's first take a closer look at just what gravitational waves are like. We'll do this by comparing gravitational waves with a more familiar form of radiation: electromagnetic waves.

There is a basic difference between electromagnetic waves and gravitational waves: gravity has only one charge, and it is always positive. Electromagnetism, on the other hand, has two charges—one positive and one negative—and as a result, an electrical charge can never be accumulated in large quantities. When an electric charge builds up in a thunder cloud, for example, lightning discharges it. In this way, electricity is self-neutralising. For the same reason, atoms almost always have the same number of protons and neutrons, so they remain electrically neutral.

Gravity is quite different: it can accumulate indefinitely. Although the gravitational force on a proton is 10^{40} (forty powers of ten) times smaller than the electrical force, it is possible to accumulate so much mass that gravity overwhelms electromagnetism: for example, in the collapse of a neutron star. In fact, there are situations where the power of gravitational waves from a single source can be comparable with the total electromagnetic output power of all the stars in the Universe!

The simplest explanation of gravitational waves is that they are ripples in the curvature of space-time. This view, while accurate, is not particularly helpful for understanding their properties. However, from a basic knowledge of gravity, you can deduce some of them. One important property of gravity is this: at a distance, the gravity of a spherical body like the Earth is the same as it would be if all the mass was concentrated in a single point. In other words, the gravity at the surface of the Earth is exactly the same as if all the mass was concentrated in a single point at its centre. While the spontaneous collapse of the Earth is unlikely, such a thing can happen to a star, such as when a black hole is created. In either case, if there is no change in the gravity when the Earth or a star is squashed into a point, then there is no change in curvature of space-time and no gravitational waves.

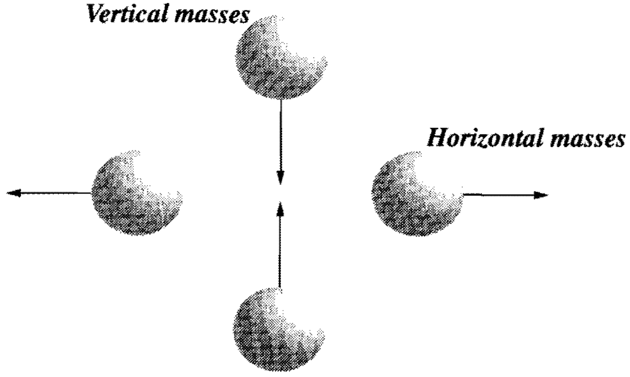
Let's try another approach to making gravitational waves. Again, let's start with the analogy of electromagnetic waves. If you take two spheres and apply a high voltage between them, it forces a bit of positive charge to the left and a negative charge to the right. Now reverse the battery terminals. If you do this fast enough, and repeatedly, you can make the electric charge oscillate between the two spheres: the process would then create electromagnetic waves. The motion of a positive charge to the left is indistinguishable from that of a negative charge to the right, and so this type of oscillator is equivalent to having a single charge moving back and forth. This is called a dipole oscillation and is the characteristic form for electromagnetic waves.

To do the same thing with gravity, you would move a single mass back and forth. That sounds easy enough—push a mass away from you then pull it back. But no! Action equals reaction: you can only move one mass to the left if another mass recoils to the right. In everyday life, we are not aware of this because we are securely attached to the ground and the recoil of the entire planet is infinitesimal, but jumping off a small boat afloat on a lake produces a more visible effect—a recoil that can lead to waves of a different kind! Because the recoil of the two motions cancels out, we cannot have a dipole motion. In fact what we get is a quadrupole motion: the spacing between the two masses changes, but the centre of mass remains unchanged. The change in the mass distribution changes the gravity, or the space-time curvature. Since the quadrupole oscillation changes the curvature of space-time, it should produce quadrupole gravitational waves.

Since a quadrupole oscillation produces an outgoing ripple of space-time, it only seems fair that such a gravitational wave would produce the same quadrupole oscillation in a pair of suspended masses. In other words, if the motion of a mass or masses out in space produces gravitational waves, it only seems fair that those gravitational waves produce motion in a distant mass—say, a gravitational wave detector here on Earth. A passing gravitational wave would cause a pair of suspended masses to move alternately apart and together.

As you may have guessed, this is a clue to how gravitational waves can be detected. What would happen if we had four masses suspended, forming a vertical diamond figure? If a gravitational wave passed through, the horizontal masses might at first move farther apart. What do the vertical masses do? They won't move farther apart as well, since this would be like our collapsing sphere, and we've already decided that the symmetry of a collapsing sphere rules out the production—and detection—of gravitational waves. In fact, the vertical pair moves closer together. As the horizontal pair begin to move closer together again, the vertical pair start to move farther apart. On and on the cycle would go, for as long

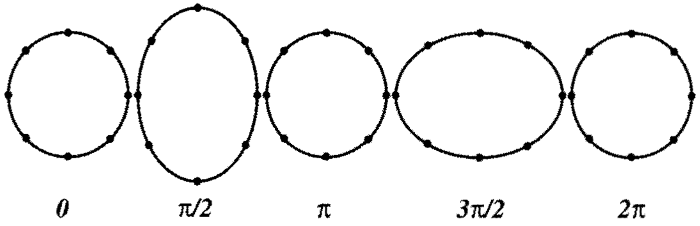
Figure 5.1 Masses suspended in a diamond shape.



as the gravitational waves were passing through. If the four masses were replaced by a solid ring, the ring would flatten into an oval alternately in one direction and then in the opposite direction, squeezed and stretched, stretched and squeezed.

There's another aspect of gravitational waves we ought to look at before going any further, and that's polarisation. Polarisation describes the direction of oscillation in a wave. Electromagnetic waves can be polarised to oscillate in two directions, which you can think of as horizontal and vertical. Television signals are usually transmitted in a horizontal polarisation—that is why our TV antennas are flat. Polaroid sunglasses allow only one direction of oscillation to pass through, thus cutting out half of the light. In the case of gravitational waves, think about those four masses suspended in a diamond configuration. When a gravitational wave which is oscillating vertically comes through, it makes those masses move further apart and closer together. If a second wave came through hot on the heels of the first, but this time oscillating horizontally, the effect would be essentially the same: the vertical masses would move further apart while the horizontal masses moved closer together, and vice versa.

Figure 5.2 Add four more masses and this is what you get.

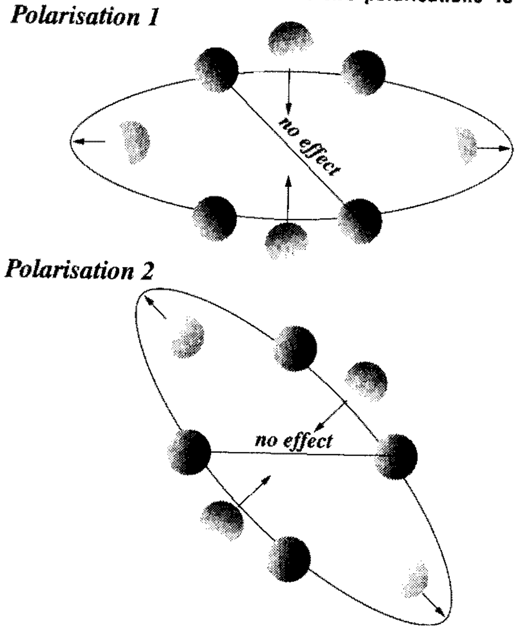


Let's add four more masses to this arrangement, so they create a square within the diamond. When either of the two gravitational waves passes through, the relative spacing of these new masses remains the same. Even though the vertical and horizontal masses oscillate like crazy, the diagonally placed masses remain stationary. The only way to make them move is to rotate the gravitational wave by 45 degrees, so that it's now alternately squeezing and stretching the diagonal masses. But wait, now the vertical and horizontal pair are stationary! Clearly, gravitational waves have two polarisations 45 degrees apart.

From a simple starting point, we have deduced the basic structure of gravitational waves: quadrupole oscillation of masses creates waves, and these waves in turn create quadrupole oscillation in masses. The next question to ask is: how much energy does it take to produce a gravitational wave?

The thing to remember about space is that it is very stiff. What we mean by this is that it is very difficult to create any sort of vibration in space. If you were to stretch a long rubber band and then pluck it, you'd hear and see the vibrations—ripples—travelling along its length. If you tried to do the same thing with something

Figure 5.3 Gravitational waves have two polarisations 45° apart.



much stiffer, say a railway line, all you would do is bruise yourself trying to get it to vibrate. You know what to do. Use a good-size sledge-hammer and give the line a good bash. You don't have to be a physicist to understand what's happening here: the wave power you inject into the line depends on the mass of the hammer, the frequency with which you hit it, and it also depends inversely on the stiffness of the line (the stiffer it is, the less wave energy generated).

Now, you can think of the two masses we used to create gravitational waves as the equivalent of a sledge-hammer. In this case, the gravitational wave power that you produce depends on the masses, their frequency of oscillation, and on the elasticity of space-time. If the

masses are close together, you would expect the gravity of the pair to be similar to the gravity of a single mass. Hence small changes in spacing between the masses will have less effect on space-time curvature. So the gravitational wave power produced should also increase with the spacing of the masses, and, of course, with the amplitude of their oscillation.

Putting all this together, we can say that:

$$\text{Gravitational wave power} = \frac{\text{mass} \times \text{spacing} \times \text{amplitude} \times \text{frequency}}{\text{stiffness of space–time}}$$

This result shows the main feature of gravitational waves: they are weak, not because there aren't some very violent processes in the cosmos, but because the wave power is divided by the enormous elastic stiffness of space-time (about 10^{43}).

The most effective source of gravitational waves is one where the spacing between the masses changes from nearly zero to some maximum value. Two masses joined by a spring would oscillate in this way, but two masses joined by a rigid rod and spun like a band leader's baton is much better. Viewed side-on, such a rotating dumbbell appears to expand and contract from maximum to zero. There's a problem, however. There's a limit to how fast something can rotate before it flies apart under centrifugal force. Even a steel ball-bearing will explode if it rotates faster than a million times a second. This rotational speed limit makes things look quite depressing for anyone planning to send signals via gravitational waves.

Undaunted, let's try to devise the best possible gravitational wave source. We want to take two large, joined masses and rotate them as fast as possible. We've seen, though, that the bar that joins them will break at high speeds, and so it's better to use a solid elongated mass. Let's be ambitious and use 1000 tonnes of steel, or even better a 10 000-tonne nuclear submarine. If we mounted the submarine on a turntable and rotated it to just short of its breaking point—say 10 times a second—how much gravitational wave energy would we produce? Sadly, not very much at all: about 10^{-24} watts. To put this into perspective, the smallest ant walking fast up a wall uses 10^{-7} watts, a billion billion times more energy than the gravitational waves produced by our rotating submarine. And the ant's efforts are 10 billion times smaller than an average family car.

Such small amounts of energy aren't new to astronomers. Radio astronomers, for example, are quite familiar with the feeble energy they find in studying the radio universe. Yet radio telescopes have one major advantage: they can collect almost all of the energy in the incoming radio waves. A gravitational wave detector on the scale of the submarine, however, could detect only a tiny fraction of the energy originally transmitted (even if all the gravitational waves passed by it).

But there are much larger oscillating masses out there in space. Binary stars, for example, behave like sun-sized dumbbells. They consist of close-spaced pairs of stars whizzing around each other. Does the existence of these objects improve our chances of detecting gravitational waves? To calculate the strength of an astrophysical source of gravitational waves, all you have to do is put the numbers into Einstein's quadrupole formula, much like the one we derived above. We could apply it to ordinary binary star systems, where the stars orbit each other as fast as once a day. In our Galaxy there are millions of such systems which, as a whole, should produce a background of random ripples in space-time at a very low frequency of one cycle per day. Each half of the binary system creates a full in-and-out motion of the masses, as seen edge on, so the gravitational wave frequency is double the orbital period. This means that the gravitational wave frequency is in the range of one

cycle every few hours to one cycle every few days. Each individual source is very weak and even the combined effect of all of them would be completely undetectable by Earth-based detectors, where disturbances such as temperature variations and tides cause relatively enormous variations on the same time scale as the signal. As a result, gravitational waves from these sources are completely swamped, although, as we will see later, they may be detected from space.

This depressing situation was understood by Einstein. From the 1920s to the 1960s, relativists believed that gravitational waves were of academic interest only. In 1961, the distinguished British astrophysicist Herman Bondi wrote: 'Gravitational waves are not only unfamiliar, even by name, but are distinctly unlikely to be observed.'

The only solutions were to build better detectors, or to come up with a better source of gravitational waves. In fact, both of these needs would be fulfilled in the 1960s. Decades earlier, predictions had been made for bizarre objects of unbelievable density: neutron stars and black holes. Neutron stars are a superdense ball of neutrons 20 to 30 kilometres in diameter, which yet contain the mass of an entire star. With such strong gravity, the escape velocity from a neutron star is very high—about 10 per cent the speed of light. In isolation, neutron stars are unlikely to be strong sources of gravitational waves, despite the fact that some of them rotate hundreds of times each second. However, they are often in mutual orbit either with an 'ordinary' star or with another neutron star, closely fulfilling the needs of our rotating mass source of gravitational waves. Not only do neutron stars rotate extremely fast, but they are also capable of very fast orbital motions. To cap it off, they are exceptionally good clocks. In short, neutron stars are a relativist's dream come true. Black holes are a close relative of pulsars, and their formation is also believed to be a possible source of gravitational waves. These strange objects are the result of the entire mass of a star (or more) collapsing into a single point of infinite density. Around it is a region of space from which not even light can escape.

Before we look at how neutron stars and black holes can produce gravitational waves and, in the case of neutron stars, prove the existence of gravitational waves, we'll take a detour and explore their origins—itself a possible source of gravitational waves.